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An Overview of Fuzzy Entropy- Some Non-Parametric Generalizations and Applications

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Abstract—*This research article presents a basic overview of the fuzzy entropy and the ideas related to fuzzy logic. The notions related to the fuzzy logic are also discussed in this paper. We present some non-parametric generalizations of the fuzzy entropy measures. Further some applications of fuzzy logic are also discussed.*

Keywords: *fuzzy logic, fuzzy entropy measures, non-parametric generalization.*

1. INTRODUCTION

The introduction of fuzzy set theory was given by Lotfi A . Zadeh, [1] a professor of electrical engineering with the University of California at Berkeley in the year 1965. The publication of the seminal paper by Zadeh [1] was a major breakthrough in the field of design systems. The fundamental fact regarding the fuzzy set theory is that it is the special generalization of crisp set theory but what makes fuzzy set theory different is the softness of boundaries. The backbone of fuzzy set theory is the membership function or membership grade. Over the last six decades, fuzzy set theory has been given a lot of attention and as a result there is hardly any area where its applications are not known may it be medical diagnosis, decision theory, economics, genetics, engineering, robotics, etc.

2. FUZZY SET AND MEMBERSHIP FUNCTION:

A fuzzy set is a set containing elements with varying membership degrees. This is different from classical sets in which elements have full membership in that set (i.e., their membership is assigned a value of 1). Elements of membership are mapped to a universe of membership values using a function theoretic form. The function maps elements of a fuzzy set into a real value belonging to the interval [0,1]. Mathematically a fuzzy set 'A' is defined as;

$$A=\left[\left(x_i,\mu_A(x_i)\right):x_i\in X,\mu_A(x_i)\in[0,1]\right]$$

Where $\mu_A(x_i)$ is a membership function which gives the degree of belongingness to the element 'x_i' then 'A'. In case, $\mu_A(x_i)$ = 0 or $\mu_A(x_i)$ =1 for all 'x_i' then 'A' is called a crisp set.

In simpler terms, membership function is characterized by $\mu_A(x_i)$ that maps all the members in set X to a number between 0 and 1.

3. NOTIONS RELATED TO FUZZY SETS:

- Containment: if $A\subset B\Leftrightarrow \mu_A(x_i)\le\mu_B(x_i)\forall x_i\in X$
- Equality: if $A=B\Leftrightarrow \mu_A(x_i)\forall(x_i)\in X$
- Complement: if A^c is complement of $A\Leftrightarrow \mu_{A^c}(x_i)=(1-\mu_A(x_i))\forall x_i\in X$
- Union: if $A\cup B$ is union of A and $B\Leftrightarrow \mu_{A\cup B}(x_i)=\max\{\mu_A(x_i),\mu_B(x_i)\}\forall x_i\in X$
- Intersection: if $A\cap B$ is intersection of A and $B\Leftrightarrow \mu_{A\cap B}(x_i)=\min\{\mu_A),\mu_B(x_i)\}\forall x_i\in X$
- Product: if AB is product of A and B $\Leftrightarrow \mu_{AB}(x_i)=\mu_A(x_i)\mu_B(x_i)\forall x_i\in X$

  Sum: if $A+B$ is sum of A and $B\Leftrightarrow\mu_{A+B}(x_i)=\mu_A(x_i)+\mu_B(x_i)-\mu_A(x_i)\mu_B(x_i)\}\forall x_i\in X$

4. NON-PARAMETRIC GENERALIZATIONS OF FUZZY ENTROPY MEASURES:

Fuzzy entropy is an important concept for measuring fuzzy information. It can be defined as the measure of a quantity of fuzzy information gained from a fuzzy set or fuzzy system is known as fuzzy entropy. It should be noted that the meaning of fuzzy entropy is quite different from the classical Shannon[17] entropy because no probabilistic concept is needed in order to define it, it is because of the fact that fuzzy entropy contains vagueness and ambiguity uncertainties, while

Shannon [17] entropy contains the randomness uncertainty (probabilistic).fuzzy entropy measures are being applied in many fields like speech processing, bioinformatics, image processing, feature selection, pattern recognition, fuzzy aircraft control etc.

De Luca and Termini [4] have introduced the following axiomatic structure of the measure of fuzzy entropy corresponding to Shannon [17] entropy measure in the year 1972.

$$H(A) = -\sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_A))\log(1-\mu_A(x_i)\right]$$

Following are some other non-parametric generalizations of the fuzzy entropy measures.

Kaufmann [5] in (1980) introduced the non-parametric generalization of fuzzy entropy measure as:

$$H(A) = -\frac{1}{\log n}\sum_{i=1}^{n}\Phi_A(x_i)\log\Phi_A(x_i)$$

In the year 1983 another generalization was given by Ebanks [6] as:

$$H(A) = \sum_{i=1}^{n} g(\mu_i)$$

In the year 1986 Kosko [7] contributed like this:

$$H(q, A) = d^q(A, A^{near}) / d^q(A, A^{far})$$

Bhandari and Pal [9] presented another generalization in the year 1993 as:

$$H_e(A) = \frac{1}{n(\sqrt{e}-1)}\sum_{i=1}^{n}[\mu_A(x_i)e^{(1-\mu_A(x_i))} + (1-\mu_A(x_i))e^{\mu_A(x_i)} - 1]$$

Pal et al [10] worked a lot in this field and came up with the following generalizations:

$$H_*(A) = k\sum_{i=1}^{n} g(\mu_i), k \in R^+, \hat{g}(t) = f(t)f(1-t)$$

$$g(t) = \hat{g}(t)\min_{0\le t\le 1}\{\hat{g}(t)\}, f \text{ concave \& increasing}$$

$$H_+(A) = k\sum_{i=1}^{n} g(\mu_A), k \in R^+, \hat{g}(t) = f(t) + f(1-t)$$

$$g(t) = \hat{g}(t) - \min_{0\le t\le 1}\{\hat{g}(t)\}, f \text{ concave.}$$

5. APPLICATIONS OF FUZZY LOGIC:

5.1 Medicine

Uncertainty and imprecision are associated with the diagnosis of almost all diseases. A single symptom can be associated with different ailments and on the other hand several ailments present in a patient can interact and interfere with the usual description of any of the diseases. One single disease can reflect differently depending upon the strength of patient, climatic conditions, food intake e.t.c. the other sources of uncertainty involved in the field of medicine are; the medical history of the patient, usually it may not be supplied by the patient himself/herself, the physical examinations carried out usually obtain objective data, but in some cases the boundry between normal and pathological is not sharp. Also the results of laboratory and other diagnostic tests are also subject to some kind of mistakes.

Fuzzy logic is the best possible means to deal with imprecision and uncertainty as it makes the use of partial truth values i.e; between true and false, yes and no, one and zero etc.

Some examples showing that fuzzy logic crosses many disease groups are as follows:

i. To analyze diabetic neuropathy and to detect early diabetic retinopathy.
ii. To predict the response to treatment with citalopram in alcohol dependence.
iii. To improve decision making in radiation therapy.
iv. To control hypertension during anesthesia.
v. To calculate volumes of brain tissue from magnetic resonance imaging (MRI), and to analyze functional MRI data.

5.2 Bioinformatics

Bioinformatics is the knowledge based on computer analysis of biological data. This data may be of the information stored in the genetic code, statistics of patient, experimental results etc. Nowadays enormous amount of data regarding the structure and function of the biological molecules and sequences is being generated by genome-sequencing projects, DNA microarrays and its related technologies. Biological data analysis is being applied to protein structures, polynucleotide etc.

This huge amount of data may carry uncertainty and impreciseness with it and hence the need of fuzzy logic and fuzzy technology is felt here. Some examples where the need of fuzzy logic are felt in bioinformatics are as follows:

i. DNA sequencing using genetic fuzzy systems.
ii. Clustering genes from microarray data.
iii. Studying the difference between polynucleotides.

iv. To increase the flexibility of protein motifs.
v. To analyze gene expression data.
vi. To predict proteins sub cellular locations from their dipeptide composition using fuzzy k- nearest neighbors algorithm.

5.3 Chemical engineering

It is the branch of engineering dealing with the application of physical science (e.g; chemistry and physics), and life sciences (e.g; biology, microbiology and biochemistry) with mathematics, to the process of converting raw materials or chemicals into more useful or valuable forms. Modern chemical engineering is concerned with new valuable materials and techniques like biomedical engineering, nanotechnology and fuel cells. In recent years, computational intelligence has been used to solve many complex problems by developing intelligent systems. Fuzzy logic is a powerful tool for decision-making systems, such as expert and pattern classification systems. Some chemical processes also involve fuzzy set theory. Following are some areas in the chemical engineering where the fuzzy logic is being employed:

i. Piping risk assessment
ii. Safety analysis
iii. Furnace control
iv. Modeling of the fluidized catalytic cracking unit of a petrochemical refinery.
v. Classification of product qualitative.
vi. Separation process
vii. Gas stream
viii. Oil stream
ix. Food produce
x. Bio reactor
xi. Reactor control

5.4. Civil engineering

Civil engineering in today's world faces many complex and non-linear problems like earth quake effects, behavior of tall buildings, wind effects, dynamic systems, material behaviors, uncertainties of soil behavior etc.

Following are some areas in the civil engineering where the fuzzy logic makes its presence:

i. Analysis of pipe networks through fuzzy approach.
ii. Site layout planning using non structural fuzzy decision support system.
iii. Multiple layer fuzzy evaluation for existing reinforced concrete bridges.
iv. Fuzzy genetic algorithm for optimization of steel structures.
v. Bounds of structural response for all possible loading combinations.

5.5. Image processing

Fuzzy image processing plays an important role in representing uncertain data. Benefits of fuzzy image processing are many and various techniques are able to manage the vagueness and ambiguity efficiently and deal with imprecise data.

In fuzzy image processing following areas make the use of fuzzy logic:

i. Fuzzy color credibility approach to color image filtering.
ii. Vision intelligence for farming using fuzzy logic optimized genetic algorithm and artificial neural network.
iii. Fuzzy logic approach to numerical water level gauge.
iv. A fuzzy data fusion method for improved motion estimation.
v. Fuzzy rules and GIS in three dimensional prediction.

6. CONCLUSION

In this paper we have presented a brief review of fuzzy logic and its related terms. Non-parametric generalizations of fuzzy entropy measures are also presented. Further we present a list of some applications which make use of fuzzy logic and fuzzy entropy.

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